Comment on “The statistics wars and intellectual conflicts of interest” by D. Mayo

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I enjoyed Prof. Mayo's comment in Conservation Biology (Mayo, 2021) very much, and agree enthusiastically with most of it. Here are my key takeaways and reflections.

I agree with Prof. Mayo that error probabilities (or error rates) are essential to consider: if you don’t give thought to what the data would be like on the assumption that your theory is false, you are likely reinforcing confirmation bias rather than establishing the validity of your theory. As Mayo argues, “stress-testing” models and results is crucial.

I also agree with Mayo that banning “bright lines” is not helpful. Some applications really require a decision to be made or a conclusion to be drawn. Should a species get specific legal protection, or not? Should a pipeline be built through a particular area, or not? Should drilling be permitted in a particular area, or not? Should there be economic incentives for an agricultural management intervention that is purported to sequester carbon, or not? Should a jurisdiction prohibit harvesting wild foods from urban ecosystems for safety concerns, or not? While such decisions are not made solely on the basis of P-values, banning bright lines may preclude making decisions in a principled, reproducible way that controls error probabilities.

Conversely, no threshold for significance, no matter how small, suffices to prove an empirical claim. As Fisher wrote:

[No isolated experiment, however significant in itself, can suffice for the experimental demonstration of any natural phenomenon; for the “one chance in a million” will undoubtedly occur, with no less and no more than, its appropriate frequency, however surprised we may be that it should occur to us. In order to assert that a natural phenomenon is experimentally demonstrable we need, not an isolated record, but a reliable method of procedure. In relation to the test of significance, we may say that a phenomenon is experimentally demonstrable when we know how to conduct an experiment which will rarely fail to give us a statistically significant result. (Fisher, 1935, emphasis added.)]

In replication lies empirical truth. While “strict” replication may be impossible in ecology—“you cannot step in the same river twice” (Heraclitus)—experiments, surveys, and other data collection campaigns can be repeated, at least approxi-
mately. If minor changes in circumstances alter the conclusions unpredictably, arguably the findings are “scientific observations” but perhaps not “biology”; the domain to which findings generalize determines the scientific discipline (see, e.g., Stark, 2018). If a result does not generalize to any other time or place, it is not clear that it has scientific utility.

I also agree with Prof. Mayo’s thesis that abandoning $P$-values exacerbates moral hazard for journal editors, although there has always been moral hazard in the gatekeeping function. Absent any objective assessment of the agreement between the data and competing theories, publication decisions may be even more subject to cronyism, “taste,” confirmation bias, etc.

Throwing away $P$-values because many practitioners don’t know how to use them is like banning scalpels because most people don’t know how to perform surgery. Those who would perform surgery should be trained in the proper use of scalpels, and those who would use statistics should be trained in the proper use of $P$-values.

In my opinion, the main problems with $P$-values are: faulty interpretation, even of genuine $P$-values; use of nominal $P$-values that are not genuine $P$-values; and perhaps most importantly, testing statistical hypotheses that have no connection to the scientific hypotheses (see below).

A $P$-value is the observed value of any statistic whose probability distribution is dominated by the uniform distribution when the null is true. That is, a $P$-value is the observed value of any measurable function $T$ of the data that doesn’t depend on any unknown parameters, and for which—if the null hypothesis is true—$\Pr\{T \leq x\} \leq x$ for all $x \in [0, 1]$. Reported “nominal” $P$-values often do not have that defining property.

One reason a nominal $P$-value may not be a genuine $P$-value is that calculating $T$ may involve many steps, including data selection, model selection, test selection, and selective reporting. Practitioners often ignore all but the final step in calculating the nominal $P$-value. That is, in reality, $T$ is generally the composition of many functions $T_n \circ T_{n-1} \circ \cdots \circ T_2 \circ T_1(\cdot)$, but often only the final step $T_n(\cdot)$ is considered in calculating the nominal $P$-value. If what is done to the data involves selection, conditioning, cherry-picking, multiple testing, stopping when things look good, or similar things, they all need to be accounted for, or the result will not be a genuine $P$-value.

In my experience, perhaps the most pernicious errors in the use of $P$-values in applications are Type III errors: answering the wrong question by testing a statistical null hypothesis that has nothing to do with the scientific hypothesis, aside from having some words in common. A statistical null hypothesis needs to capture the science, or testing it sheds no light on the matter. For example, consider a randomized controlled trial with a binary treatment and a binary

\footnote{There are other definitions of Type III errors, such as “correctly rejecting the null, but for the wrong reason.”}
outcome. The scientific null is that the treatment does not have an effect (either subject by subject, or on average across the subjects in the trial). A typical statistical null is that the responses to treatment and placebo are all IID $N(\mu, \sigma^2)$. The scientific null does not involve independence, normal distributions, or equality of variances. A genuine $P$-value for the statistical null does not say much about the scientific null: it is the chance that the difference in observed means would be as large or larger than observed if the responses were independent random samples from the same normal distribution.

Here is a more nuanced example: do wind turbines with longer rotors kill more raptors than turbines with shorter rotors, per kW of generating capacity? An analyst might posit a model for raptor-turbine collisions, say a zero-inflated Poisson regression model with coefficients for rotor length, peak RPM, capacity, some site characteristics, average and peak windspeed, season, mean atmospheric visibility, the estimated size of the relevant raptor population, and other covariates. The statistical hypothesis is then that the coefficient of rotor length in the model is zero. Even if one computes a genuine $P$-value for that statistical hypothesis, what does it have to do with the original scientific question? Why is a Poisson model appropriate? (For example, the Poisson model assumes that collisions are independent, but some raptors migrate in flocks.) Even if a Poisson model were appropriate, why should the log of the rate of the Poisson distribution depend linearly (or in any other parametric way) on those covariates, and not on anything else? The $P$-value is the chance that the estimated coefficient would be as large as it was if collisions followed a Poisson model with the specified dependence of log rate on exactly those covariates, and the true coefficient of rotor length in that model were zero.

I close with a comment regarding likelihood-based tests, which are mentioned briefly in Prof. Mayo’s commentary. Here, I disagree with her. There are indeed tests that depend only on likelihoods or likelihood ratios—and that allow optional stopping when “the data look good”—but that nonetheless rigorously control the probability of a Type I error. Wald’s sequential probability ratio test (Wald, 1947) is the seminal example, but there are a host of other martingale-based methods that give the same protections and give “anytime-valid” $P$-values.

References