Journal of the American Statistical Association

ISSN: 0162-1459 (Print) 1537-274X (Online) Journal homepage: https://www.tandfonline.com/loi/uasa20

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To cite this article: Allan Birnbaum (1970) On Durbin's Modified Principle of Conditionally, Journal of the American Statistical Association, 65:329, 402-403, DOI: 10.1080/01621459.1970.10481090

To link to this article: https://doi.org/10.1080/01621459.1970.10481090

© Journal of the American Statistical Association
March 1970, Volume 65, Number 329
Theory and Methods Section

On Durbin's Modified Principle of Conditionality

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The conditionality (ancillarity) concept is discussed with particular reference to Durbin's proposed reformulation (C') of Birnbaum's formulation (C) and Fisher's original formulations [6, 7]. It is pointed out that (C') is not in general suitable for applications in the usual sense, since it fails to eliminate all ambiguities of choice of the conditional frame of reference in which standard statistical methods may be applied. The alternative formulations of conditionality are discussed in relation to sufficiency, likelihood, and related methodological problems.

Continuing critical analysis of concepts of statistical evidence is most welcome since previous analyses have left "the important problem of an adequate non-Bayesian concept of statistical evidence . . . not only unresolved but in a positively anomalous state" [3, p. 301]. The present note is restricted to some specific points raised by Durbin [5], since my current general views (which differ appreciably from those in [2]) have been given in [3, 4]. For brevity we adopt without restatement the definitions and notations of Durbin's note and my article [2].

Concerning the significance of Durbin's analysis and reformulation of conditionality, we note that apart from conditionality and its consequences we now also have the censoring concept due to Pratt [8, 9], discussed also in [3]). This seems at least as plausible and attractive a concept as conditionality (and simpler), and it implies the principal part of the likelihood axiom (L). Hence even outright rejection of conditionality would not suffice to eliminate our perplexities about concepts of statistical evidence.

In any case, to one who (like this writer) has found the conditionality idea attractive, but its apparent consequence the likelihood axiom (L) less plausible and applicable, it seems interesting to consider a reformulation of conditionality which may not imply too much of the content of (L). However Durbin's formulation (C'), although weaker than (C), is nevertheless too strong (implies too much of the content of (L)) to be compatible with standard (non-Bayesian) statistical concepts and techniques. The basis for this statement is that (C') does not in general lead to unambiguous determination of a conditional model. For example, (C') allows all of the non-unique ancillaries in the examples in [1, 2], since those ancillaries are functions of minimal sufficient statistics.

On the level of general concepts, it seems not enough to point out, as Durbin does quite plausibly, that (C') "seems to be consistent with what most statisticians mean by conditional inference." Probably most statisticians who have found *both* sufficiency and conditionality plausible as *general* ideas consider

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them as complementary and compatible, and will be ready to take second thoughts about details of formulations when they see that the order of applying the two concepts can make a difference in some formulations but not in others. (They can also consider (C''), in which simplification is made first by conditionality, then by sufficiency.)

Both sufficiency and conditionality have been most concretely illustrated and applied through rules of substitution of simpler for more complex models of experiments (in which the remaining steps of analysis and interpretation of statistical evidence have been left more or less open, in theoretical discussions, and handled by standard non-Bayesian techniques in applications). However the identification of such general concepts of statistical evidence with corresponding substitution rules is evidently an oversimplification, which Durbin's formulation does not avoid. In contrast, (C) and (S) are equivalence relations, expressing respective general concepts, and in principle justifying substitutions in two directions. (Here no effects can follow from changes of "order of application.") Indeed this general character of each concept "seems to be consistent with what most statisticians mean by" the *ideas* of sufficiency and conditionality, as they have been discussed and frequently applied since Fisher introduced them (without claiming definitive analysis of their significance and implications for theory and applications).

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