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The conditionality (ancillarity) concept is discussed with particular reference to Durbin's proposed reformulation (C') of Birnbaum's formulation (C) and Fisher's original formulations [6, 7]. It is pointed out that (C') is not in general suitable for applications in the usual sense, since it fails to eliminate all ambiguities of choice of the conditional frame of reference in which standard statistical methods may be applied. The alternative formulations of conditionality are discussed in relation to sufficiency, likelihood, and related methodological problems.

Continuing critical analysis of concepts of statistical evidence is most welcome since previous analyses have left "the important problem of an adequate non-Bayesian concept of statistical evidence . . . not only unresolved but in a positively anomalous state" [3, p. 301]. The present note is restricted to some specific points raised by Durbin [5], since my current general views (which differ appreciably from those in [2]) have been given in [3, 4]. For brevity we adopt without restatement the definitions and notations of Durbin's note and my article [2].

Concerning the significance of Durbin's analysis and reformulation of conditionality, we note that apart from conditionality and its consequences we now also have the censoring concept due to Pratt [8, 9], discussed also in [3]). This seems at least as plausible and attractive a concept as conditionality (and simpler), and it implies the principal part of the likelihood axiom (L). Hence even outright rejection of conditionality would not suffice to eliminate our perplexities about concepts of statistical evidence.

In any case, to one who (like this writer) has found the conditionality idea attractive, but its apparent consequence the likelihood axiom (L) less plausible and applicable, it seems interesting to consider a reformulation of conditionality which may not imply too much of the content of (L). However Durbin's formulation (C'), although weaker than (C), is nevertheless too strong (implies too much of the content of (L)) to be compatible with standard (non-Bayesian) statistical concepts and techniques. The basis for this statement is that (C') does not in general lead to unambiguous determination of a conditional model. For example, (C') allows all of the non-unique ancillaries in the examples in [1, 2], since those ancillaries are functions of minimal sufficient statistics.

On the level of general concepts, it seems not enough to point out, as Durbin does quite plausibly, that (C') "seems to be consistent with what most statisticians mean by conditional inference." Probably most statisticians who have found both sufficiency and conditionality plausible as general ideas consider

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them as complementary and compatible, and will be ready to take second
thoughts about details of formulations when they see that the order of applying
the two concepts can make a difference in some formulations but not in others.
(They can also consider \(C''\), in which simplification is made first by condition-
ality, then by sufficiency.)

Both sufficiency and conditionality have been most concretely illustrated and
applied through rules of substitution of simpler for more complex models of ex-
periments (in which the remaining steps of analysis and interpretation of statis-
tical evidence have been left more or less open, in theoretical discussions, and
handled by standard non-Bayesian techniques in applications). However the
identification of such general concepts of statistical evidence with correspond-
ing substitution rules is evidently an oversimplification, which Durbin's formu-
lation does not avoid. In contrast, \((C)\) and \((S)\) are equivalence relations, ex-
pressing respective general concepts, and in principle justifying substitutions
in two directions. (Here no effects can follow from changes of "order of applica-
tion." ) Indeed this general character of each concept "seems to be consistent
with what most statisticians mean by" the ideas of sufficiency and condition-
ality, as they have been discussed and frequently applied since Fisher introduced
them (without claiming definitive analysis of their significance and implications
for theory and applications).

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