V. Johnson

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The example is the Normal testing case of J. Berger and Sellke, but they compare it to a one-tailed test of H_0: $\mu = 0$ vs. H_1: $\mu = \mu_1 = \mu_{max}$ (entirely sensibly in my view). We abbreviate H_1 by H_{max}. Here the likelihood ratio $\text{Lik}(\mu_{max})/\text{Lik}(\mu_0) = \exp[z^2/2]$); the inverse is $\text{Lik}(\mu_0)/\text{Lik}(\mu_{max}) = \exp[-z^2/2]$. I think the former makes their case stronger, yet you will usually see the latter. (I record their values in a Note[9]). What is μ_{max}? It's the observed mean $\overline{x}$, the place most "favored by the data." In each case we consider $\overline{x}$ as the result that is just statistically significant at the indicated P-value, or its standardized z form.

SIST p. 260: Table 4.2

Table 4.2 Upper Bounds on the Comparative Likelihood

P-value: one-sided	z_α	$\text{Lik}(\mu_{\max})/\text{Lik}(\mu_0)$
0.05	1.65	3.87
0.025	1.96	6.84
0.01	2.33	15
0.005	2.58	28
0.0005	3.29	227

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Valen Johnson (2013a,b) offers a way to bring the likelihood ratio more into line with what counts as strong evidence, according to a Bayes factor. He begins with a review of "Bayesian hypotheses tests." "The posterior odds between two hypotheses H_1 and H_0 can be expressed as"

$$\frac{\Pr(H_1|\boldsymbol{x})}{\Pr(H_0|\boldsymbol{x})} = \mathrm{BF}_{10}(\boldsymbol{x}) \times \frac{\Pr(H_1)}{\Pr(H_0)}.$$

Like classical statistical hypothesis tests, the tangible consequence of a Bayesian hypothesis test is often the rejection of one hypothesis, say H_0, in favor of the second, say H_1. In a Bayesian test, the null hypothesis is rejected if the posterior probability of H_1 exceeds a certain threshold. (Johnson 2013b, pp. 1720–1)

SIST p. 262: Table 4.3

Table 4.3 V. Johnson's implicit alternative analysis for T+: H_0: $\mu \le 0$ vs. H_1: $\mu > 0$

P-value one-sided	z_α	$Lik(\mu_{max})/Lik(\mu_0)$	μ_{max}	$Pr(H_0 \vert x)$	$Pr(H_{max} \vert x)$
0.05	1.65	3.87	$1.65\sigma\sqrt{n}$	0.2	0.8
0.025	1.96	6.84	$1.96\sigma\sqrt{n}$	0.128	0.87
0.01	2.33	15	$2.33\sigma\sqrt{n}$	0.06	0.94
0.005	2.58	28	$2.58\sigma\sqrt{n}$	0.03	0.97
0.0005	3.29	227	$3.3\sigma\sqrt{n}$	0.004	0.996
	$\sqrt{(2 \log k)}$	$exp\left(\frac{z_\alpha^2}{2}\right)$	$z_\alpha \sigma\sqrt{n}$	$1/(1+k)$	$k/(1+k)$

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We perform our two-part criticism, based on the minimal severity requirement. The procedure under the looking glass is: having obtained a statistically significant result, say at the 0.005 level, reject H_0 in favor of H_{max}: $\mu = \mu_{max}$. Giving priors of 0.5 to both H_0 and H_{max} you can report the posteriors. Clearly, (S-1) holds: H_{max} accords with $\bar{x}$ – it's equal to it. Our worry is with (S-2). H_0 is being rejected in favor of H_{max}, but should we infer it? The severity associated with inferring μ is as large as μ_{max} is

$$\Pr(Z < z_{\alpha}; \mu = \mu_{max}) = 0.5.$$