Discussion: On Arguments Concerning Statistical Principles

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(i) *Statistical inference after Neyman–Pearson*. Statistical inference as an alternative to Neyman–Pearson decision theory has a long history in statistical thinking, with strong impetus from Fisher's research; see, for example, the overview in Fisher (1956). Some resulting concerns in inference theory then reached the mathematical statistics community rather forcefully with Cox (1958); this had focus on the two measuring-instruments example and on uses of conditioning that were compelling.

(ii) *Birnbaum and logical analysis in statistical inference*. Birnbaum (1962) introduced notation for the statistical inference available from an investigation with a model and data. This gave grounds to analyze how different methods or principles might influence the statistical inference. As part of this he discussed how sufficiency, likelihood and conditioning could differentially affect statistical inference. Much of his discussion centered on the argument from conditioning and sufficiency to likelihood, but a primary consequence was the attention attracted to conditioning and its role in inference. While this interest in conditioning was substantial for those concerned with the core of statistics, it has more recently been neglected or overlooked. Indeed, some recent texts, for example, Rice (2007), seem not to acknowledge conditioning in inference or even the measuring-instrument example.

(iii) *Mayo and statistical principles*. Mayo should be strongly commended for reminding us that the principles and arguments of statistical inference deserve very serious consideration and, we might add, could have very serious consequences (Fraser, 2014). Her primary focus is on the argument (Birnbaum, 1962) that the principles sufficiency and conditionality lead to the likelihood principle. This may not cover some recent aspects of conditioning (Fraser, Fraser and Staicu, 2010), but should strongly stimulate renewed interest in conditioning.

(iv) *Contemporary inference theory*. Many statistical models have continuity in how parameter change affects observable variables or, more specifically, how parameter change affects coordinate quantile functions, the inverses of the coordinate distribution functions. This continuity in its global effect is widely neglected in statistical inference. If this effect on quantile functions is accepted and used in the inference procedures, then in wide generality there is a well-determined conditioning (Fraser, Fraser and Staicu, 2010). And likelihood analysis then offers an exponential model approximation that is third-order equivalent to the given model, and this in turn provides third-order inference for any scalar component parameters of interest. Thus, the familiar conditioning conflicts are routinely avoided by acknowledging the important model continuity.

(v) *What is available?* The conditioning just described leads routinely to p-value functions $p(\psi)$ for any scalar component parameter $\psi = \psi(\theta)$ of the statistical model. A wealth of statistical inference methodology then immediately becomes available from such p-value functions. For example, a test for a value ψ_0 is given by the p-value $p(\psi_0)$, a confidence interval by the inverse $(\hat{\psi}_{\beta/2}, \hat{\psi}_{1-\beta/2}) = p^{-1}(1 - \beta/2, \beta/2)$ of the p-value function, and a median estimate by the value $p^{-1}(0.5)$. But quite generally the needed p-value functions are not available from a likelihood function alone!

(vi) *What are the implications?* If continuity is included as an ingredient of many model-data combinations, then, as we have indicated, likelihood analysis produces p-values and confidence intervals, and these are not available from the likelihood function alone. This thus demonstrates that with such continuity-based conditioning the likelihood principle is not a consequence of sufficiency and conditioning principles. But if we omit the continuity then we are directly faced with the issue addressed by Mayo.

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