

EXAMPLE SET 1: illustration using the Excel program

The curves in the Excel example depict the hypotheses in test $T+$, the one-sided Normal test of $H_0: \mu \leq 12$ (blue curve) against $H_1: \mu > 12$ (red curve).

Null (or test) hypothesis $H_0: \bar{X}$ is Normal $(\mu, \sigma / \sqrt{n})$, and $\mu = 12$.

Alternative $H_1: \bar{X}$ is Normal $(\mu, \sigma / \sqrt{n})$, and $\mu > 12$.

Each hypothesis, being statistical, assigns probabilities to the different values of X , and thus to the different values of the sample mean \bar{X} . Those are the heights of the curves. Areas under the curve are probabilities for events like $\{\bar{X} > 12.4\}$.

EXAMPLE SET 1: Throughout, keep $\sigma = 2$, and $n = 100$. So $\sigma / \sqrt{n} = 2/10$. Note: σ / \sqrt{n} is the standard deviation of \bar{X} . I will sometimes abbreviate this as σ_x because it's too hard to get an x-bar as a subscript of σ . (Ignore the lines under the curves for now, just look at the curves and the boxes on the left.)

Let's specify the test rule: the test $T+$ will reject H_0 whenever $\bar{X} \geq 12 + 2 \sigma / \sqrt{n}$

$T+$ reject H_0 if and only if $\{\bar{X} \geq 12.4\}$.

For $\{\bar{X} < 12.4\}$. "do not reject" (i.e., accept) H_0 .

$\{\bar{X} \geq 12.4\}$. is the rejection region of the test.

This test has a probability of rejecting H_0 under the assumption we are in the world of the first curve, i.e., assuming $\mu = 12$, of .023 (with a slight approximation). This is the probability of a type 1 error, alpha.

(1) $P[\{\bar{X} \geq 12.4\}; \mu = 12] = .023$

In the picture, this is shown as the turquoise area under the curve. It is also written as the 4th entry in the turquoise box on the left. The turquoise box lists: the population mean (as given in the null of the test, which we keep at 12) the population SD, σ , which we keep at 2, the sample size, n , which (for Example Set 1) we keep at 100. Below that is alpha, which we keep at .023 for this example set.

The only thing to be changed is the alternative (red) curve. The point is to see the shifts in the probability test $T+$ rejects the null under different assumptions for the alternative. Even though the alternative is still $H_1: \mu > 12$, we will compute the probability of rejection under different point values μ_1 greater than 12.

(2) $P\{\bar{X} \geq 12.4\}; \mu = \mu_1$ = The Power of test $T+$ (to reject H_0) when the true value of μ is μ_1

We can abbreviate this as $POW(\mu_1)$.

The grey box on the left shows the sample mean \bar{X} . For Example set 1, we will not change this. We assume \bar{X} just makes it to the cut-off for rejection, i.e., 12.4.

$$\bar{X} = 12.4.$$

The gray box also shows the corresponding p-value which is $\sim .023$

Example 1.1

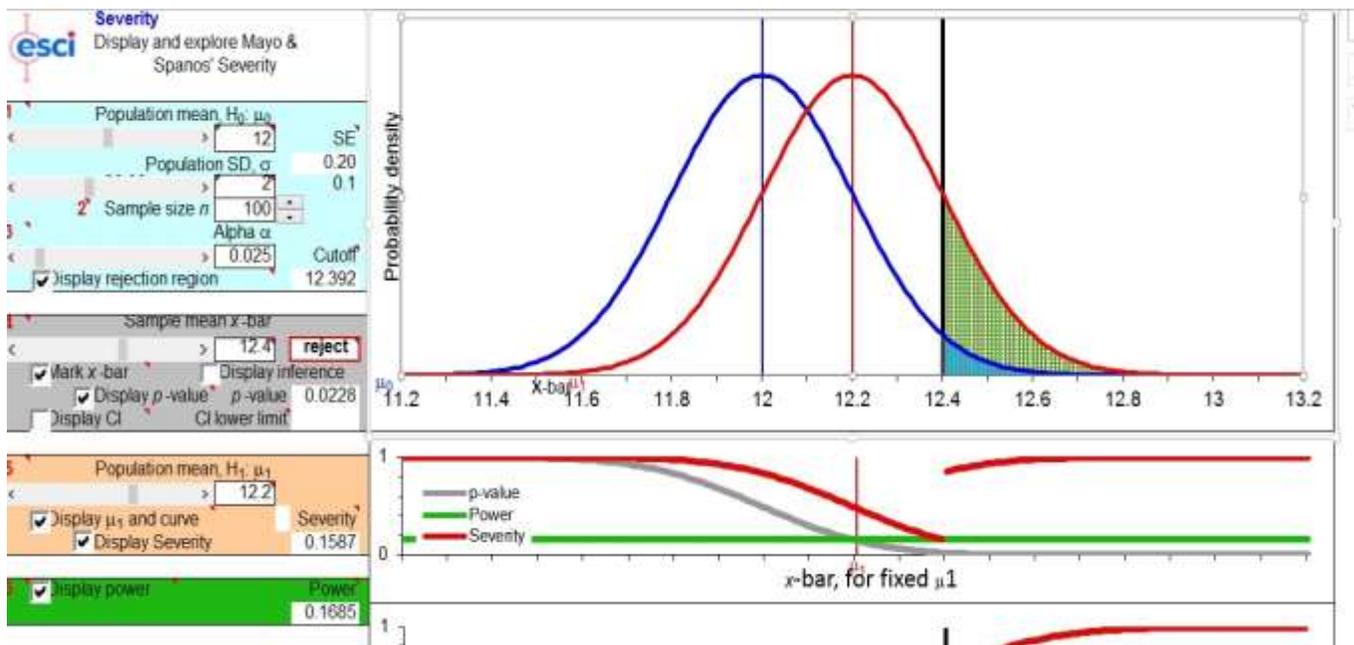
Finally we get to the good part. The figure below shows the case where the alternative is $\mu = 12.2$. That is, it sets $\mu_1 = 12.2$ in the bottom, peachy-colored, box on the left.

We would like to know

(2a) $P\{\bar{X} \geq 12.4\}; \mu = 12.2$ = The probability test T+ would reject H_0 when the true value of μ is 12.2.

POW(12.2).

What is it? It is the green area under the red curve, which is .16. Why do we look at the area under the red curve, rather than under the blue curve? Because to the right of “;” in (2a) is $\mu = 12.2$. It’s asking for a calculation assuming we are in the hypothesized red world. So imagine you’re in the red curve world where $\mu = 12.2$: the probability $\{\bar{X} \geq 12.4\}$ is .16.



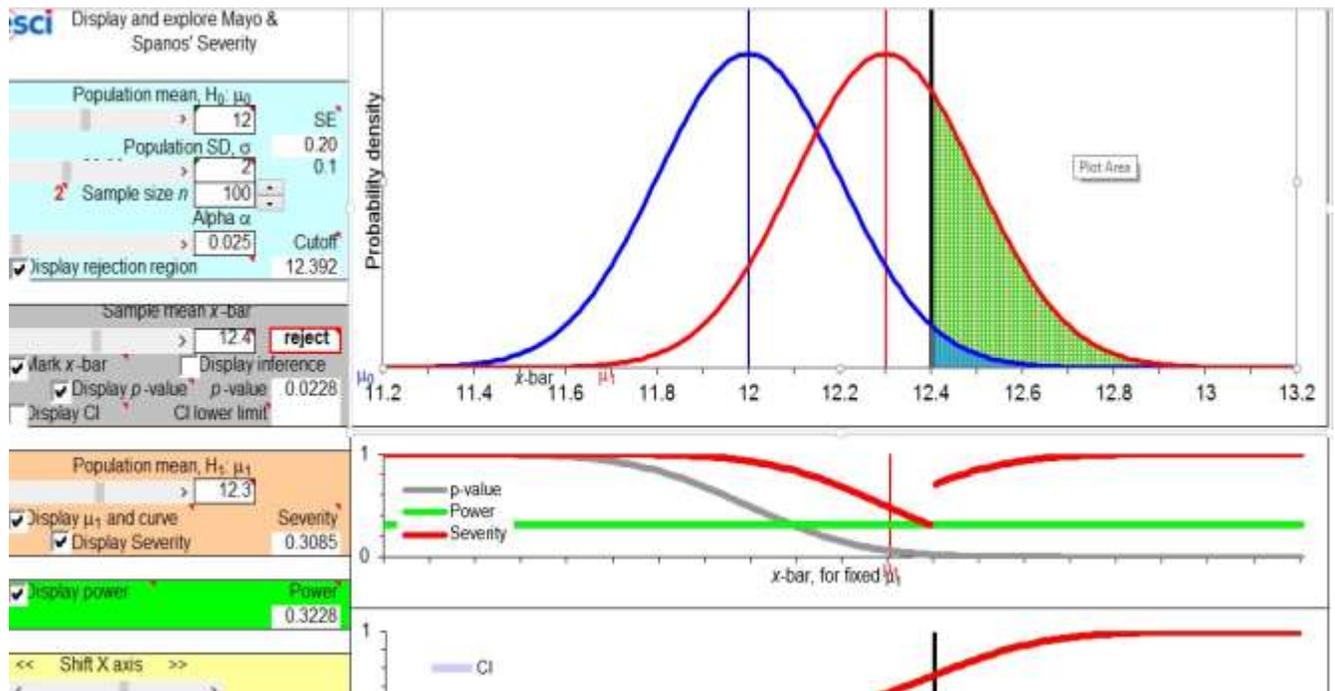
Example 1.2

Now consider the alternative $\mu = 12.3$.

(2b) $P\{\{\bar{X} \geq 12.4\}; \mu = 12.3\}$ = The probability test T+ would reject H_0 when the true value of μ is 12.3.

=.3

= POW(12.3).



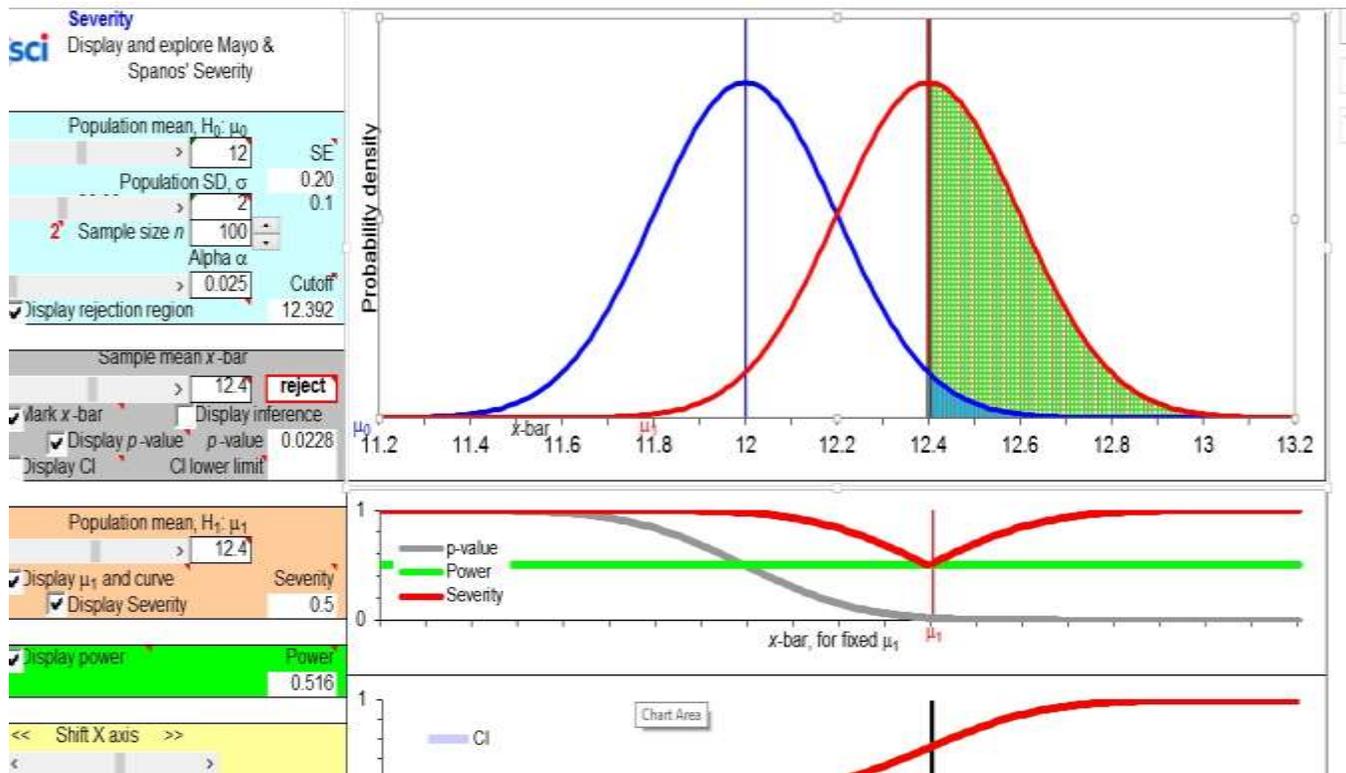
Example 1.3

Now consider the alternative $\mu = 12.4$.

(2c) $P\{\bar{X} \geq 12.4; \mu = 12.4\}$ = The probability test T_+ would reject H_0 when the true value of μ is 12.4.

=.5

= POW(12.4).



Example 1.4

Now consider the alternative $\mu = 12.5$.

$$(2d) P\{\bar{X} \geq 12.4\}; \mu = 12.5] = .7$$

$$= \text{POW}(12.5).$$

